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Designing tuned mass dampers via static output feedback: a numerical approach

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SUMMARY

The paper presents a numerical approach to the problem of determining the design parameters of a vibration absorber to minimize the effect of disturbing forces. The approach is also applicable to multi-input systems such as civil engineering structures. The resulting devices are appealing since they can be implemented passively and are more economic and reliable than active control devices. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: vibration absorber; tuned mass damper

1. INTRODUCTION

Machines are often subjected to disturbance forces which inhibit their operation and they need to be protected from the disturbance, particularly when the disturbance frequency is close to the resonance frequency of the machine. Common example are machines tools such as sanders and saws, washing machines, compressors, motors and even electric razors. Elimination of the disturbing forces would be the first choice to cure the situation, however, this is seldom practical or even possible.

An alternative approach is to use a vibration absorber or tuned mass damper, invented by Frahm in 1909. A vibration absorber consists of a small spring–mass system which is added to a machine to prevent vibration of the machine in spite of the disturbing force. These passive vibration control devices are also used on civil engineering structures to damp the excitation from wind or seismic disturbances. The problem facing the designer is to determine values for the absorber parameters. An analysis of the design of vibration absorbers for single-degree-of-freedom systems is given by Den Hartog in 1956 [1]. A more recent discussion can be found in most vibration books (see, for example, [2]).

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The simplest situation is that of a machine (main mass) modelled without damping. Then it is well known [1, 2] that an absorber consisting of a mass m_a and stiffness k_a will eliminate the vibration if the absorber frequency is set equal to the frequency of the disturbing force. There are two main limitations to this design. First, if the frequency of the disturbing force shifts even slightly, then the effect of the absorber is diminished and it can even amplify the vibration rather than suppress it. Furthermore, the excitation is seldom a pure sinusoid at a single frequency, but often contain secondary harmonics, so that the main mass is being excited at multiple frequencies. Second, all systems have some damping and the damping in the main system realistically cannot be neglected.

To overcome the first limitation, damping is added to the absorber to improve the effective bandwidth of the absorber, i.e. to increase the range of frequencies for which the absorber dissipates energy. For such situations, Den Hartog [1] has developed formulas for "optimal" values of the absorber spring, mass and damping constants. However, these formulas assumed no main mass damping and hence are not applicable to realistic situations where the main system includes damping.

If damping is included in the main system, the approach of Den Hartog [1] used to compute the optimal parameters can no longer be used because of the complicated nature of the resulting equations. Consequently, one must resort to a computer study of the response of the system for many different combinations of the absorber parameters (essentially a three-dimensional gridding procedure) and create tables of optimum values (see, for example, [3]). Obviously, not all possible values of the absorber parameters can be included in the tables and only a small collection of possible choices are available to the designer. These results are applicable to single-degree-of-freedom system (SDOF); similar tables are not available for multi-degree-of-freedom systems (MDOF).

Active control techniques have been suggested as a method for vibration suppression. While this approach can be shown to theoretically suppress vibrations, the resulting controllers have little practical use in many situations because the implementation of the controllers require that actuators and sensors be added to the system. The additional hardware gives rise to cost and reliability problems and active controllers may not be accepted in industrial applications. We will show below that by using a novel formulation of the output feedback control problem, the resulting controller can implemented passively, i.e. through the use of a standard spring–mass–damper absorber system. In effect, this approach can be used to design absorber parameters for single-degree-of-freedom systems with damping, obviating the need for look-up tables or extensive simulation in 3-D parameter space, and, most importantly, is readily extended to multi-degree-of-freedom systems such as civil engineering structures. These passive devices are appealing from both an economic and reliability viewpoint.

2. THE OUTPUT FEEDBACK PROBLEM

Consider the system

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1}$$

$$z = C_1 x + D_{12} u \tag{2}$$

$$y = C_2 x \tag{3}$$

where $x \in R^n$ is the state vector, $w \in R^q$ is the disturbance input, $z \in R^r$ is the controlled output, $u \in R^m$ is the control input and $y \in R^p$ is the measured output.

A control $u = Ky$ is called a $\gamma -$ *stabilizing output feedback control* if

(i) $A + B_2 K C_2$ is stable, and

(ii) $\| T_{zw}(s) \|_\infty < \gamma$,

where $T_{zw}(s)$ is the transfer function from w to z and γ is the disturbance attenuation. The infinity norm is defined as

$$\| T_{zw}(s) \|_\infty = \sup_\omega \bar{\sigma} [T_{zw}(j\omega)]$$

where $\bar{\sigma}$ denotes the maximum singular value. If the disturbance attenuation is γ, then, with zero initial conditions,

$$\int_0^\infty z^T(t) z(t)\, dt \leqslant \gamma^2 \int_0^\infty w^T(t) w(t)\, dt$$

Furthermore, γ is an estimate of the peak-to-peak gain of the system in the sense that as γ becomes small, then the gain δ in $\|z(t)\|_\infty < \delta \|w(t)\|_\infty$ becomes small, where for vector v, $\|v(t)\|_\infty = \max_i \sup_{t \geqslant 0} |v_i(t)|$.

Several approaches for solving the difficult problem of determining the static output feedback gain K have appeared in the literature. One approach is based on solving two non-linear matrix inequalities [4, 5]; another [6] involves an iteration technique. These techniques can be effective, but we were unsuccessful in applying them to the problems posed here.

Furthermore, the gain matrix K in our control $u = Ky$ may have certain elements set equal to zero so input i is independent of output measurement j. The procedures in References [4–6] do not readily accommodate such constraints. Problems with elements of K constrained to be zero will always arise in multi-input problems and one of the main attractions of our approach is the ability to analyse such problems.

Let $\mathscr{K}$ be the set of $m \times p$ matrices that have zeros in the desired positions. For $K \in \mathscr{K}$, the transfer function from w to z is

$$T_{zw}(s) = (C_1 + D_{12} K C_2) [sI - (A + B_2 K C_2)]^{-1} B_1$$

The optimization problem is to choose $K \in \mathscr{K}$ to minimize $\| T_{zw}(s) \|_\infty$ subject to the resulting closed-loop system being stable, i.e. the maximum real part of the eigenvalues of $A + B_2 K C_2$ are negative. This problem is readily solved using the Optimization Toolbox from Matlab. Magnitude constraints on the elements of K also can readily be incorporated into the program.

In the next section we show how the design of the mass, spring and damping constants of the vibration absorber can be carried out by solving this output feedback problem.

3. INTRODUCTORY EXAMPLES

Consider the vibration absorber system shown in Figure 1. The equations of motion are

$$m_1 \ddot{x}_1 + (k_1 + k_d) x_1 + (c_1 + c_d) \dot{x}_1 - k_d x_d - c_d \dot{x}_d = w \tag{4}$$

$$m_d \ddot{x}_d + k_d (x_d - x_1) + c_d (\dot{x}_d - \dot{x}_1) = 0 \tag{5}$$

$$z = x_1 \tag{6}$$

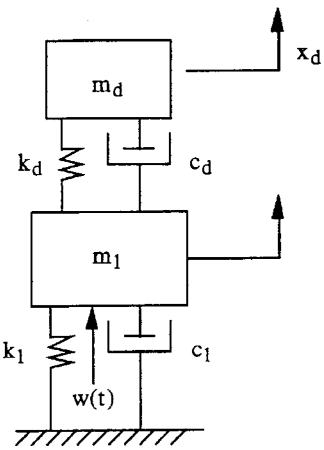

Figure 1. Passive vibration absorber.

Many mechanical systems are modelled by these equations, see for example, References [1, 2]. The parameters of the primary system — m_1, k_1, c_1 — are taken as given. The mass ratio $\mu = m_d/m_1$ is fixed, usually $0 < \mu < 0.25$.

The simplest case is when $c_1 = 0$, no damping in the main mass system. Then, from Reference [1], the parameters which minimize $\|T_{zw}(s)\|_\infty$ are

$$\frac{\omega_d}{\omega_1} = \frac{1}{1+\mu}, \quad \frac{c_d}{2\sqrt{m_d k^d}} = \sqrt{\frac{3\mu}{8(1+)^3}} \tag{7}$$

where

$$\omega_d = \sqrt{\frac{k_d}{m_d}}, \quad \omega_1 = \sqrt{\frac{k_1}{m_1}} \tag{8}$$

are the natural frequencies of the main mass and absorber, respectively. With these parameter choices,

$$\|T_{zw}(s)\|_\infty = \frac{1}{k_1}\sqrt{\frac{2+\mu}{\mu}}$$

For example, if $m_1 = 1$, $k_1 = 1.25$ and $\mu = 0.25$, then $k_d = 0.2$, $c_d = 0.098$ and $\|T_{zw}(s)\|_\infty = 2.4$.

We now present an output feedback approach to obtain these parameters. Again consider $c_1 = 0$. First, rewrite Equations (4) and (5) in state-space form

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(k_1+k_d)/m_1 & k_d/m_1 & -c_d/m_1 & c_d/m_1 \\ k_d/m_d & -k_d/m_d & c_d/m_d & -c_d/m_d \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix} w \tag{9}$$

$$z = [1 \;\; 0 \;\; 0 \;\; 0]x$$

where $x = [x_1 \; x_d \; \dot{x}_1 \; \dot{x}_d]^T$ is the state vector, w is the disturbance input, and z is the controlled output. Since the purpose of the absorber is to limit the magnitude of the main system's displacement, the controlled output, z, is set equal to x_1. The main mass parameters, m_1 and k_1, are given and the parameters of the absorber, m_d, k_d and c_d, are selected to decrease the vibration of the main mass. Usually, m_d is specified as a fixed percentage of the main mass, $m_d = \mu m_1$.

Now consider the following system with control input:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix} w + \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ -1/m_d \end{bmatrix} u \tag{10}$$

$$y = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} x, \quad z = [1 \quad 0 \quad 0 \quad 0] x$$

where m_1, k_1, m_d, x and w are as in Equation (3), u is the control input and y is the measured output. Choosing a static output feedback controller $u = Ky$ to stabilize the system and minimize the infinity norm from w to z for this problem is equivalent to choosing the parameters k_d and c_d, i.e. $K = [k_d \; c_d]$. With this static output feedback controller, the closed-loop system is the same as system (4)–(5). Thus, the problem of choosing the absorber parameters is equivalent to a static output feedback problem! All of the machinery available for solving static output feedback problems can be brought to bear on the problem of designing vibration absorbers. Although we are designing an active controller, the controller can be implemented passively.

Solving the optimization problem of Section 2 for the parameters leads to $k_d = 0.1995$, $c_d = 0.1265$ and $\| T_{zw}(s) \|_\infty = 2.38$. Thus, we obtain the classical result via the static output feedback problem approach.

As a second example to illustrate the effectiveness of our approach, we consider the same problem with damping included in the model of the main mass ($c_1 \neq 0$). We analyse three cases with different parameter values, as shown in Table I. The output feedback approach leads to nearly the same values as those obtained by exhaustive search in Reference [3], supporting the viability of the output feedback approach. Here γ denotes $[T_{zw}(s)]_\infty$.

A similar formulation of the vibration absorber problem is presented in Reference [7]. There, however, they first obtain a dynamic controller and then use an *ad hoc* method to approximate it by a constant gain feedback corresponding to the damping and stiffness coefficients. Here we

Table I.

Mass ratio μ	Main system damping c_1	Warburton [3]			Output feedback		
		k_d	c_d	γ	k_d	c_d	γ
0.01	0.02	0.0098	0.0013	11.37	0.0098	0.0013	11.28
0.01	0.2	0.0093	0.0014	3.967	0.0093	0.0013	3.93
0.1	0.02	0.0819	0.0338	4.27	0.0818	0.341	4.24

obtain the coefficients directly. Also, it is not clear if the approximate method in Reference [7] can be extended to multi-input problems. In Sections 5 and 6, we show that our approach can be used for the multi-input problems that arise when several TMDs are used.

4. APPLICATION TO A SEISMIC EXCITED BUILDING

In this section, we consider a seismic excited building with identical storeys [8]. The mass, stiffness and damping coefficients for each storey are $m_i = 1$ metric ton, $k_i = 980$ kN/m and $c_i = 1.407$ kN-sec/m for $i = 1, 2, 3$. With a state vector consisting of the interstorey drifts and velocities, the equations of motion are

$$\dot{x} = Ax + B_1 w \tag{11}$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_1 = [0\ 0\ 0\ \ -1\ 0\ 0]^{\mathrm{T}} \tag{12}$$

where w is the earthquake acceleration.

$$M^{-1}K = \begin{bmatrix} k_1/m_1 & -k_2/m_2 & 0 \\ -k_1/m_1 & k_2/m_1 + k_2/m_2 & -k_2/m_2 \\ 0 & -k_2/m_2 & k_2/m_2 + k_3 m_3 \end{bmatrix} \tag{13}$$

and $M^{-1}C$ can be obtained by replacing k_i with c_i in $M^{-1}K$. The controlled output z is taken as x_1 so that $z = [1\ 0\ 0\ 0\ 0\ 0]x$.

First, we consider the case where a TMD with mass $m_{\mathrm{d}} = 0.1$ m $= 0.1$ metric ton is placed on the top floor, see Figure 2. The state vector becomes $x = [x_1\ x_2\ x_3\ x_{\mathrm{d}}\ \dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_{\mathrm{d}}]^{\mathrm{T}}$, where x_{d} is the relative displacement of the TMD, and the equations of motion are as in Equations (11) and (12) with

$$M^{-1}K_4 = \begin{bmatrix} k_1/m_1 & -k_2/m_1 & 0 & 0 \\ -k_1/m_1 & k_2/m_1 + k_2/m_2 & -k_3/m_2 & 0 \\ 0 & -k_2/m_2 & k_3/m_2 + k_3 m_3 & 0 \\ 0 & 0 & -k_3/m_3 & 0 \end{bmatrix}$$

and $M^{-1}C_4$ changed correspondingly. B_1 becomes

$$B_1 = [0\ 0\ 0\ 0\ \ -1\ 0\ 0\ 0]^{\mathrm{T}}$$

Following the procedure in Section 2, the problem of choosing the TMD parameters k_{d} and c_{d} can be re-cast as an output feedback problem with

$$\dot{x} = \begin{bmatrix} 0_{4\times 4} & I_{4\times 4} \\ -M^{-1}K_4 & -M^{-1}C_4 \end{bmatrix} x + \begin{bmatrix} 0_{4\times 1} \\ -1 \\ 0_{3\times 1} \end{bmatrix} w + \begin{bmatrix} 0_{6\times 1} \\ 1/m_3 \\ -1/m_3 - 1/m_{\mathrm{d}} \end{bmatrix} u \tag{14}$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x \tag{15}$$

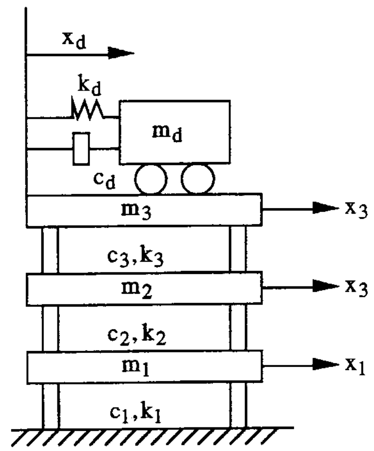

Figure 2. Three storey building with TMD on top floor.

Table II. Maximum response quantities for El Centro earthquake.

Maximum quantities	Case 1	Case 2	Case 3	Case 4
x_1 (cm)	1.33	0.51	0.69	0.68
x_2 (cm)	1.02	0.43	0.57	0.51
x_3 (cm)	0.59	0.25	0.34	0.30
$\ddot{x}_{a1}$ (m/s^2)	3.11	1.47	1.89	1.82
$\ddot{x}_{a2}$ (m/s^2)	4.74	1.99	2.37	2.12
$\ddot{x}_{a3}$ (m/s^2)	5.81	2.48	3.07	2.71

The optimization problem described in Section 2 of minimizing $\|T_{zw}(s)\|_\infty$ by choice of $u = Ky = [d_\mathrm{d}\ c_\mathrm{d}]y$ leads to the TMD parameter values of $k_\mathrm{d} = 17.03$ kN/m and $c_\mathrm{d} = 0.3855$ kN sec/cm with $\|T_{zw}(s)\|_\infty = 0.0159$.

The simulation results for the earthquake data from the E1 Centro earthquake, scaled to a peak acceleration of 0.11 g as in Reference [8] are presented in Table II.

Case 1 gives the results with no control For comparison, we include as Case 2 typical results from Reference [7] for an active controller using only a measurement of the first floor velocity, i.e. $u = K\dot{x}_1$. (In Reference [8], it is demonstrated that this measurement is the best measurement choice for a static output feedback controller based on one measurement.) Case 3 presents results for a TMD on the third floor. The response quantities with the TMD are similar to those in Case 2. But the TMD can be implemented passively and the reliability and economic problems that arise with an active controller are not present.

5. CONSTRAINED OUTPUT FEEDBACK

In this section, we analyse the situation where TMDs are placed on both the second and third floors. Since this is a multi-input problem, some of the elements of the gain matrix must be kept at

zero. The state vector becomes $x = [x_1\ x_2\ x_3\ x_{d2}\ x_{d3}\ \dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_{d2}\ \dot{x}_{d3}]^{\mathrm{T}}$ where x_{d2} and x_{d3} are the relative displacements of the TMDs on the second and third floor, respectively, and the equations of motions are as in Equations (11) and (12) with

$$M^{-1}K = \begin{bmatrix} k_1/m_1 & -k_2/m_1 & 0 & 0 & 0 \\ -k_1/m_1 & k_2/m_1 + k_2/m_2 & -k_3/m_2 & -k_{d2}/m_2 & 0 \\ 0 & -k_2/m_2 & k_3/m_2 + k_3/m_3 & k_{d2}/m_2 & -k_{d3}/m_3 \\ 0 & -k_2/m_2 & k_3/m_2 & k_{d2}/m_{d2} + k_{d2}/m_3 & 0 \\ 0 & 0 & -k_3/m_3 & 0 & k_{d3}/m_{d3} + k_{d3}/m_3 \end{bmatrix}$$

and $M^{-1}C$ is obtained by replacing k_i with c_i.

Consequently, the output feedback problem becomes

$$\dot{x} = \begin{bmatrix} 0_{5\times 5} & I_{5\times 5} \\ -M^{-1}K_5 & -M^{-1}C_5 \end{bmatrix} x + \begin{bmatrix} 0_{6\times 1} \\ 1 \\ 0_{3\times 1} \end{bmatrix} w + \begin{bmatrix} \multicolumn{2}{c}{0_{6\times 2}} \\ 1/m_2 & 0 \\ -1/m_2 & 1/m_3 \\ -1/m_2 - 1/m_{d2} & 0 \\ 0 & -1/m_{d3} - 1/m_3 \end{bmatrix} u$$

where the control is constrained to be of the form

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_{d2} & c_{d2} & 0 & 0 \\ 0 & 0 & k_{d3} & c_{d3} \end{bmatrix} y$$

and $M^{-1}K_5$ and $M^{-1}C_5$ are the same as $M^{-1}K$ and $M^{-1}C$ except the fourth and fifth columns are all zeros. Observe that the problem is now more restricted than a standard output feedback problem since some of the elements of the gain matrix must be zero.

Solving the corresponding optimization problem leads to $k_{d2} = 12.8115$, $c_{d2} = 0.212$, $k_{d3} = 18.3988$ and $c_{d3} = 0.4035$ with $\gamma = 0.01176$. Thus, compared to a single TMD on the third floor, γ is reduced by 26%. The response to the El Centro earthquake is given as Case 4 in Table II. Since there is very little improvement achieved by using two TMDs, it is questionable whether it is beneficial to add an additional TMD.

Through this type of analysis, a building designer can now decide if it is worth the added expense of including a second TMD.

6. AN ACTIVE–PASSIVE DEVICE

As a final example, we consider an innovative active control device introduced in References [9–11] and shown in Figure 3. The equations of motion are

$$m\ddot{x} + c\dot{x} + kx - c_d\dot{y} - k_d y = w$$

$$m_d(\ddot{x} + \ddot{y}) + c_d\dot{y} + k_d y - c_s\dot{z} - k_s z = -u$$

$$m_s(\ddot{x} + \ddot{y} + \ddot{z}) + c_s\dot{z} + k_s z = u$$

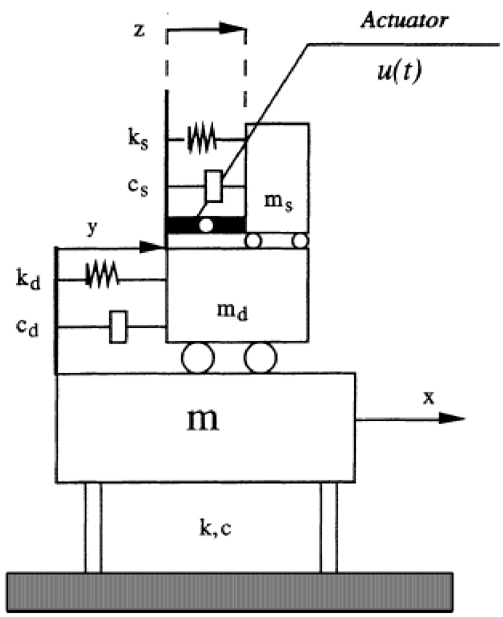

Figure 3. Active–passive device.

where $m = 16274$ kg, $k = 1.05 \times 10^6$ N/m and $c = 5.40 \times 10^3$ Nsec/m. The active control u is restricted to be of the form

$$u = G_1 \dot{x} + G_2 y$$

In general, there are six parameters for the designers to choose: k_s, c_s, k_d, c_d, G_1 and G_2. In References [10, 11], only c_d and G_2 are varied and the other parameters are fixed at $k_s = 2.25 \times 10^3$ N/m, $c_s = 1.47 \times 10^3$ Nsec/m, $k_d = 2.65 \times 10^4$ N/m and $G_1 = 1.47 \times 10^4$ Nsec/m. The free parameters are chosen to be $c_d = 5.29 \times 10^2$ Nsec/m and $G_2 = 1.42 \times 10^4$ N/m. As a measure of the effectiveness of the controller, $\max_\omega |x/F|$ for $w(t) = Fke^{j\omega t}$ is used and equals 5.72 for these parameters.

With our approach, we let all six parameters vary and solve for these parameters via a constrained output feedback problem as described in Section 2. With $u_1 = G_1 \dot{x} + G_2 y - k_s z - c_s \dot{z}$ and $u_2 = k_d y + c_d \dot{y}$, the equations of motion become

$$m\ddot{x} + c\dot{x} + kx = w + u_2$$

$$m_d(\ddot{x} + \ddot{y}) = -u_1 - u_2$$

$$m_s(\ddot{x} + \ddot{y} + \ddot{z}) = u_1$$

and the control is constrained to be of the form

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & G_1 & G_2 & 0 & -k_s & -c_s \\ 0 & 0 & k_d & c_d & 0 & 0 \end{bmatrix} [x \;\; \dot{x} \;\; y \;\; \dot{y} \;\; z \;\; \dot{z}]^{\mathrm{T}}$$

Table III. Results for active–passive device.

Parameters	Nishimura *et al.* [10, 11]	Static output feedback
m	16 274 kg	16 274 kg
k	1.04×10^6 N/m	1.05×10^6 N/m
c	5.40×10^3 Nsec/m	5.40×10^3 Nsec/m
m_d	388.6 kg	388.6 kg
k_6	2.65×10^4 N/m	2.6155×10^4 N/m
c_d	5.29×10^2 Nsec/m	0
M_s	37.8 kg	37.8 kg
k_s	2.25×10^3 N/m	1.876×10^3 N/m
C_s	1.47×10^3 Nsec/m	0.163×10^3 Nsec/m
G_1	1.47×10^4 Nsec/m	1.47×10^4 Nsec/m
G_2	1.42×10^4 N/m	0.9106×10^4 N/m
$\lvert x/F \rvert$	5.72	3.09

The results of solving the constrained static output feedback problem are displayed in Table III along with the results from References [10, 11]. In solving this problem, the parameters were restricted in magnitude by the maximum parameter values in References [10, 11]. If we increased the maximum allowable parameter values, we could also improve performance.

Using our output feedback approach and varying all six parameters, the guaranteed worst response to a sinusoidal input is reduced by 46 per cent.

7. CONCLUSIONS

We have presented a method for solving the problem of selecting the parameters for a tuned mass damper. Our approach is to replace the TMD problem with an active control problem where the control is static output feedback controller. We show that the active controller can be implemented passively as a TMD. The active controller design is carried out via an optimization problem. Several examples, including a TMD to mitigate the effect of seismic excitation on a building, are included to demonstrate the application of the results.

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